

Pudsey Waterloo Primary School

Calculation Policy



Mathematics Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Background

The 2014 Primary National Curriculum for mathematics differs from its predecessor in many ways. Alongside the end of Key Stage year expectations, there are suggested goals for each year; there is also an emphasis on depth before breadth and a greater expectation of what children should achieve. One of the key differences is the level of detail included, indicating what children should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered. In many ways, these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils' calculation skills. However, the expectation of using formal methods is rightly coupled with the explicit

requirement for children to use concrete materials and create pictorial representations – a key component of the mastery approach.

How to use the policy

This mathematics policy is a guide for all staff at Pudsey Waterloo Primary school and has been adapted from work from White Rose Maths. It is purposely set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the focus must always remain on breadth and depth rather than accelerating through concepts. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems. All teachers have been given the scheme of work from the White Rose Maths and are required to base their planning around their year group's blocks and small steps and not to move onto a higher year group's scheme work. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. The school agreed list of terminology is located at Appendix A

Content of the Policy

1. Early Years – Addition, Subtraction, Multiplication and Division.
2. Addition
3. Subtraction
4. Multiplication
5. Division
6. Mathematical Language

Early Years

Research on children's learning in the first six years of life demonstrates the importance of early experiences in mathematics. An engaging and encouraging climate for children's early encounters with mathematics develops their confidence in their ability to understand and use mathematics. These positive experiences help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence, which contribute to their future success in and out of school (Clements & Conference Working Group, 2004).

The NCTM (National Council of Teachers of Mathematics) states:

“Young learners’ future understanding of mathematics requires an early foundation based on a high-quality, challenging, and accessible mathematics education. Young children in every setting should experience mathematics through effective, research based curricula and teaching practices. Such practices in turn require that teachers have the support of policies and resources that enable them to succeed in this challenging and important work.”

They go on to highlight how early maths can support the aims of the new Curriculum 2014:



“Early childhood educators should actively introduce mathematical concepts, methods, and language through a variety of appropriate experiences. Teachers should guide children in seeing connections of ideas within mathematics as well as with other subjects, developing their mathematical knowledge throughout the day and across the curriculum. They must encourage children to communicate, explaining their thinking as they interact with important mathematics in deep and sustained ways.”

Mathematics involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures. (Statutory Framework for the Early Years Foundation Stage, DfE: 2012)

Addition

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

Foundation Stage

1. Have an understanding of what "more" means and be able to say what is one more than a given number.
2. Children begin to combine groups of objects or pictures and use concrete apparatus. 
3. Solve simple problems using fingers and introduce Numicon when appropriate.  $2 + 5 = 7$



4. Children make a record in pictures, words, Numicon shapes or symbols of addition activities already carried out.

$$\begin{array}{c} \text{🍓} \text{🍓} \\ 2 \end{array} + \begin{array}{c} \text{🍓} \text{🍓} \\ 2 \end{array} = 4$$



$$5 + 1 = 6$$

5. Children are encouraged to read number sentences aloud in different ways:

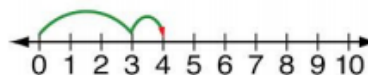
e.g. "Three add two equals 5"

"Four plus 3 makes 7"

6. Construct number sentences verbally, or by using cards to go with practical activities.

7. Number lines can be used alongside practical apparatus to solve addition calculations and word problems. Children "jump" along the number line to "count on".

$$3 + 1 = 4$$



Key Vocabulary: Games and songs can be a useful way to begin using the vocabulary involved in addition. **add, more, plus, makes, total, altogether, score, double, one more, two more, ten more**
how many more to make...? how many more is ... than ...?

Key skills for addition in Foundation Stage:

- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Use the language of 'more' and 'fewer' to compare two sets of objects.
- Find the total number of items in two groups by counting all of them.
- Say the number that is one more than a given number.
- Find one more from a group of up to five objects, then ten objects.
- In practical activities and discussion, begin to use the vocabulary involved in addition
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations

Subtraction

Foundation Stage

1. Have an understanding of what "less" means and be able to say what is one less than a given number.
2. Children begin to use objects, pictures and concrete apparatus to relate subtraction to taking away and counting how many objects are left.
3. Solve simple problems using fingers and introduce Numicon where appropriate.

 $5 - 1 = 4$

 $5 - 3 = 2$



4. Children make a record in pictures, words, Numicon shapes or symbols of subtraction activities already carried out.

10 take away 5 leaves 5

5. Children are encouraged to read number sentences aloud in different ways
e.g. **"Five subtract one leaves four"** **"Six take away 3 equals 3"**
6. Construct number sentences verbally or using cards to go with practical activities.
7. Number lines can be used alongside practical apparatus to solve subtraction calculations and word problems "jump" back to "count down" the number line.



Key Vocabulary: Games and songs can be a useful way to begin using the vocabulary involved in subtraction: e.g. Five Little Men in a Flying Saucer, Ten Green Bottles, Five Currant Buns
take, take away, leave, subtract, minus, equals, number sentence, count back, one less, two less, ten less
how many are left / left over? how many have gone? how many fewer is ... than ...?

Key skills for subtraction in Foundation Stage:

- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Use the language of 'more' and 'fewer' to compare two sets of objects.
- Say the number that is one less than a given number.
- Find one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, begin to use the vocabulary involved in subtraction
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations

Multiplication

Foundation Stage

The link between addition and multiplication can be introduced through doubling and reinforced through repeated addition of the same number.

1. Children begin with mostly pictorial representations.



How many groups of 2 are there? 3 groups of 2 = 6

2. Real equipment to count in repeated groups of the same size.

life contexts and use of practical



How many wheels are there altogether?



How much money do I have?

3. Count in twos, fives and tens, both aloud and with objects, such as Numicon or other concrete apparatus.

2, 4, 6, 8, 10, 12

4. Children are encouraged to read number sentences aloud in different ways

e.g. "Five groups of two makes ten" "Three lots of two makes six"

5. Children are given multiplication problems set in a real life context and are encouraged to visualise the problem.

e.g. "How many fingers on two hands?" "How many sides on three triangles?"



"How many legs on four ducks?"



Key Vocabulary: lots of, groups of, times, repeated addition, double, combine, twos, fives, tens

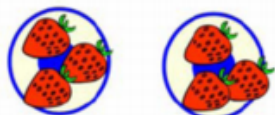
Key skills for multiplication in Foundation Stage:

- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Find the total number of items in two groups by counting all of them.
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations.

Division

Foundation Stage

1. Division can be introduced through halving or sharing an equal amount into 2 groups.



2. Children begin with mostly pictorial representations linked to real life contexts:



Grouping Model

Mum has 6 socks. She grouped them into pairs. How many pairs did she make?



Sharing Model

I have 10 sweets. I want to share them with my friend. How many will we have each?

Children need to see and hear representations of division as both grouping and sharing.

3. Children have a go at recording the calculation that has been carried out:
e.g. by drawing pictures in groups or by arranging concrete apparatus into groups.



12 shared equally by 3 is 4

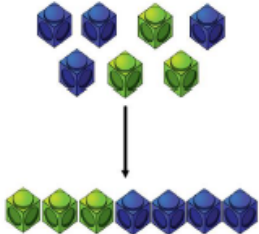
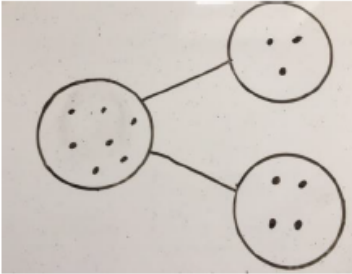
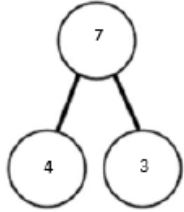
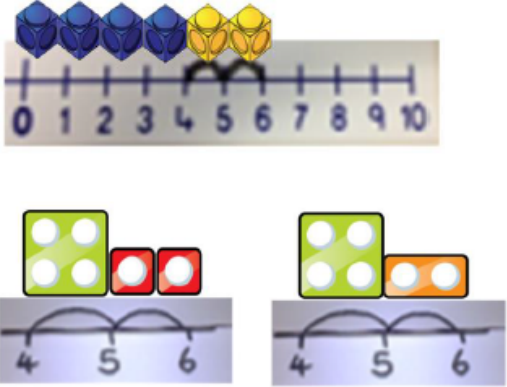
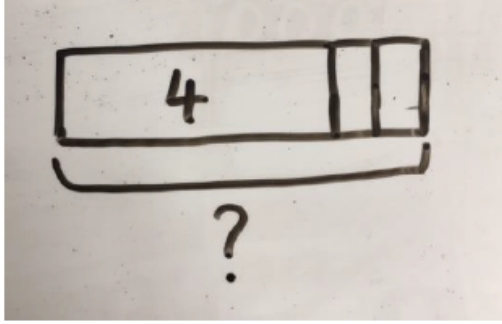
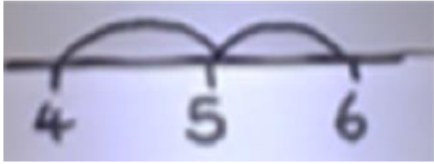
Key Vocabulary: halve, share, share equally, one each, two each, three each, group in pairs / threes / tens, equal groups of, in equal parts, left, left over

Key skills for division in Foundation Stage:

- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations.

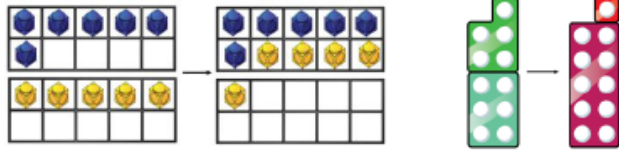
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

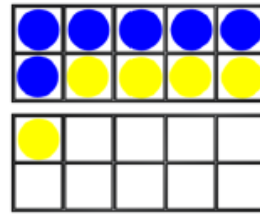
Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p> 
<p>Counting on using number lines using cubes or Numicon.</p> 	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p> 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

6 + 5



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

$6 + \square = 11$

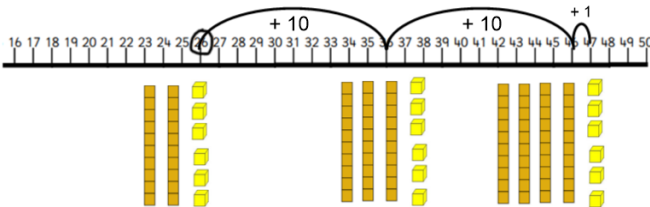
$6 + 5 = 5 + \square$

$6 + 5 = \square + 4$

TO + O using a number line

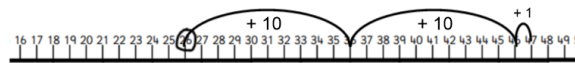
Children count on using a number line but use Base 10 or Numicon to visually see the number, adding 10 or ones each time.

26 + 21 =



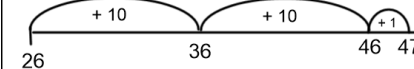
Children partition the smallest number and jump on in jumps of tens and ones using a numbered number line

26 + 21 =



Children partition the smallest number and jump on in jumps of tens and ones using a blank number line

26 + 21 =



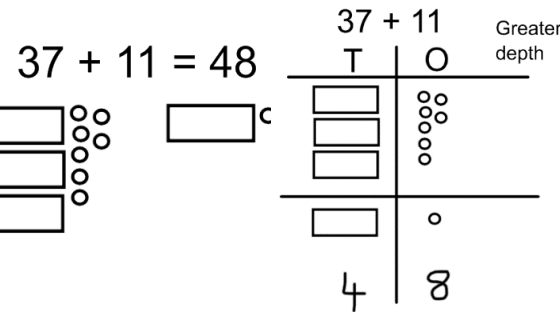
TO + O using Malteser method

Continue to develop understanding of partitioning and place value alongside base 10. This is a visual representation that helps children to imagine ten Maltesers in a packet and individual Maltesers.

31 + 6

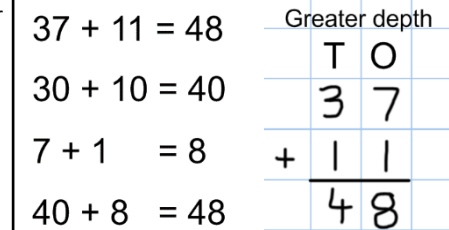


Children represent the Maltesers by drawing the packets as rectangles and circles for individual Maltesers.

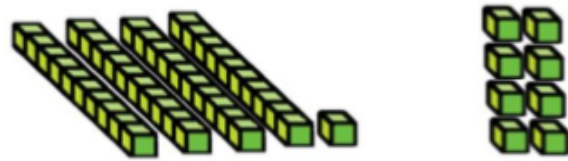


Children are adding the tens and adding the ones to find the total.

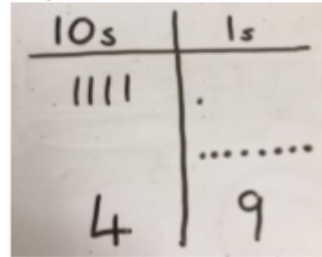
Children are developing partitioning to add.



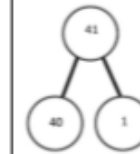
TO + O using base 10. Continue to develop understanding of partitioning and place value.
41 + 8



Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



41 + 8

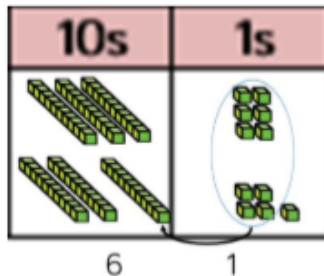


$$1 + 8 = 9$$

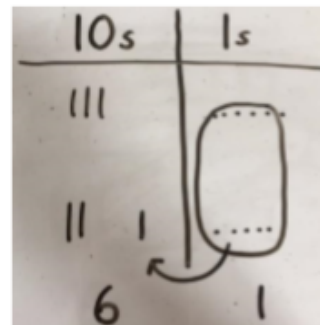
$$40 + 9 = 49$$

	4	1
+		8
	4	9

TO + TO using base 10. Continue to develop understanding of partitioning and place value.
36 + 25



Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

$$36 + 25 =$$

1 5

$$30 + 20 = 50$$

$$5 + 5 = 10$$

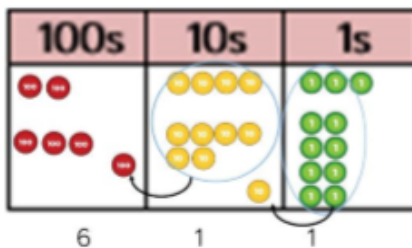
$$50 + 10 + 1 = 61$$

36

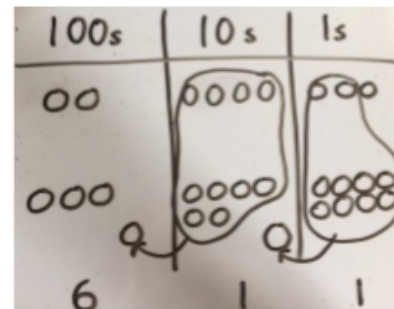
Formal method:

$$\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$$

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



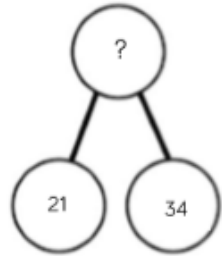
Children to represent the counters in a place value chart, circling when they make an exchange.



243

$$\begin{array}{r} +368 \\ 243 \\ \hline 611 \\ 1 \quad 1 \end{array}$$

Conceptual variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:
 In year 3, there are 21 children and in year 4, there are 34 children.
 How many children in total?

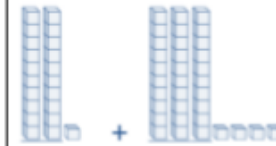
$21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

$$\boxed{} = 21 + 34$$

Calculate the sum of twenty-one and thirty-four.

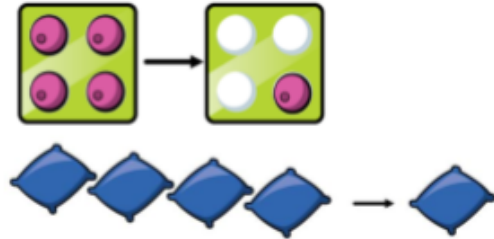
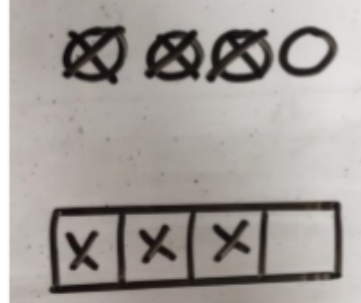
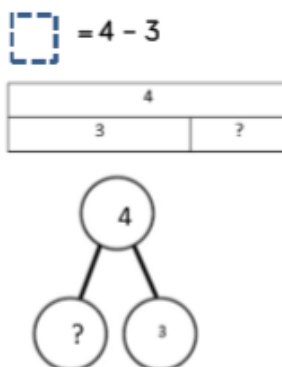

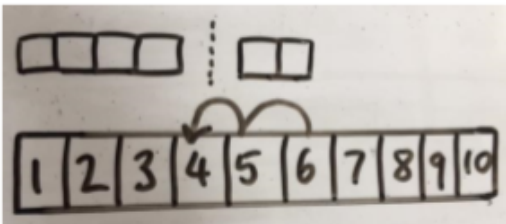
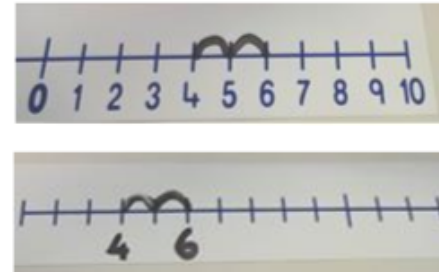


Missing digit problems:

10s	1s
● ●	●
● ● ●	?
?	5

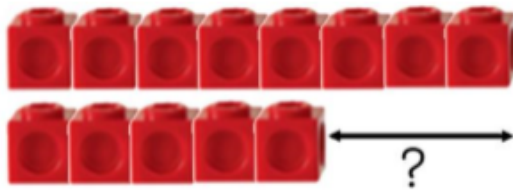
Calculation policy: Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

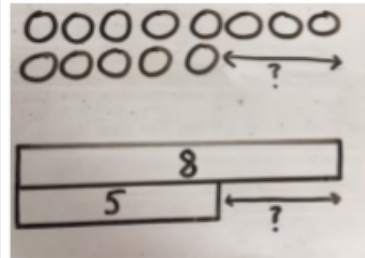
Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> 
<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p> 

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



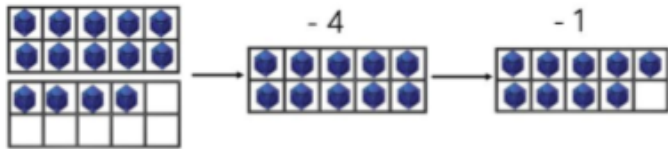
Find the difference between 8 and 5.

8 - 5, the difference is

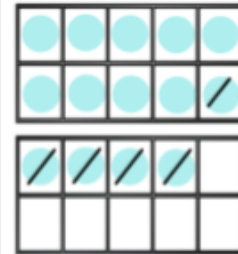
Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

14 - 5



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

$$14 - 4 = 10$$

$$10 - 1 = 9$$

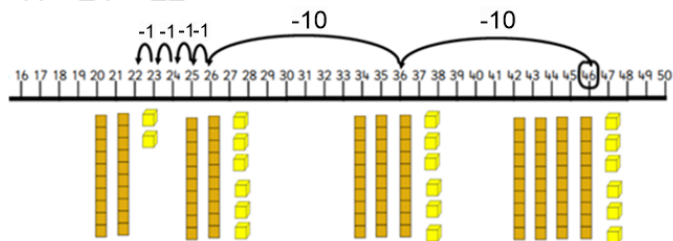
Subtraction on a number line

TO – TO

Children jump back on a number line – the jumps are above the line. Children jump back in tens and then ones.

Base ten or numicon will support the pupils visually make the link between the numbers and concrete apparatus.

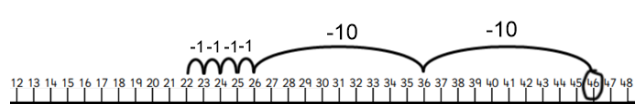
$$46 - 24 = 22$$



TO - TO

Children partition the number they are subtracting into tens and ones. Children jump back in tens and then ones on a numbered number line.

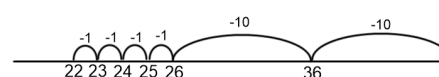
$$46 - 24 = 22$$



TO – TO

When children are secure with a number line, place value and subtracting they can move to a blank number line. Children write the whole number at the end of the number line and subtract by partitioning.

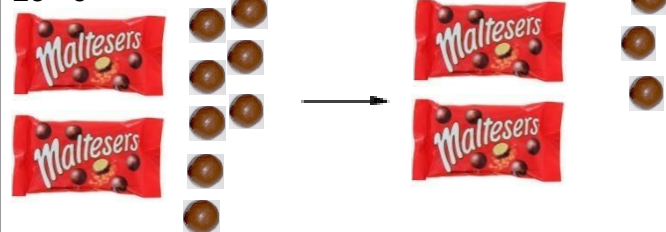
$$46 - 24 = 22$$



Subtracting using Malteser method

TO – O no exchange – children will physically take them away

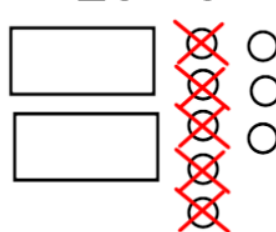
$$28 - 5$$



TO – O

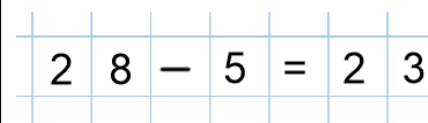
Children will draw the Maltesers and cross out the tens or ones taken away

$$28 - 5 = 23$$



TO – O

Children will count back in their head



Subtracting using Malteser method

TO – TO with exchange

Children can take away the tens but cannot take away the ones so they open a packet to reveal 10 more Maltesers. The ones can then be taken away.
45 – 28

4 5 - 2 8 = 1 7

TO – TO with exchange

Children will draw the Malteser packets. They will cross out the tens and then open a packet using a zig-zag line to show that they are opening a packet. The 10 Maltesers from the packet are then drawn out. The remaining ones are then crossed out.

45 – 28

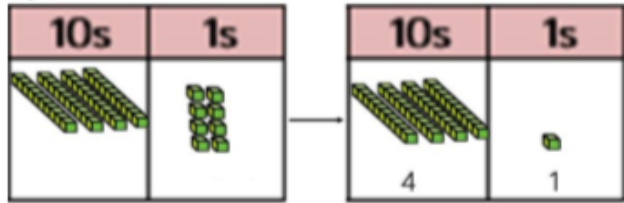
4 5 - 2 8 = 1 7

TO – TO

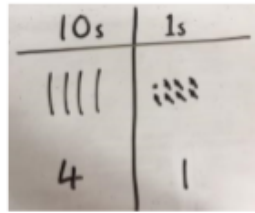
Children could subtract the tens and then subtract the ones mentally or complete a 2 part calculation e.g.

4	5	-	2	8	=	1	7
4	5	-	2	0	=	2	5
2	5	-	8		=	1	7

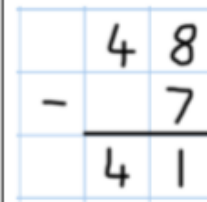
Column method using base 10.
48-7



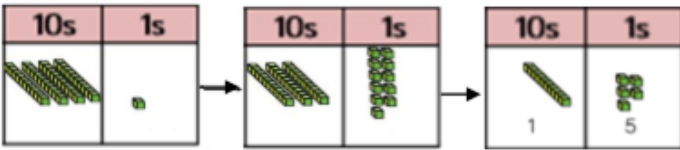
Children to represent the base 10 pictorially.



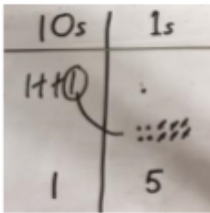
Column method or children could count back 7.



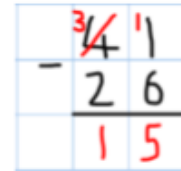
Column method using base 10 and having to exchange.
41 - 26



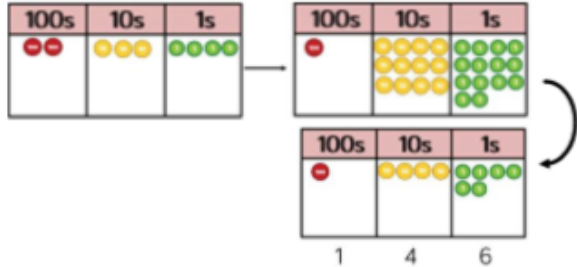
Represent the base 10 pictorially, remembering to show the exchange.



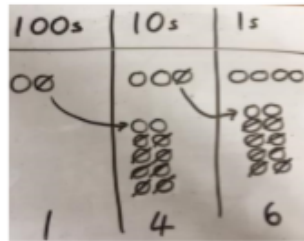
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.



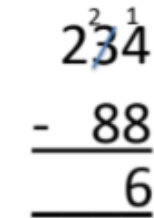
Column method using place value counters.
234 - 88



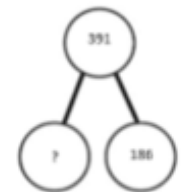
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.



Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\boxed{} = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

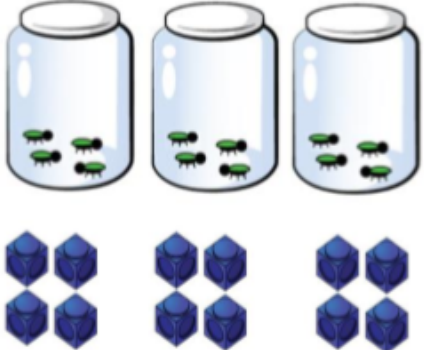
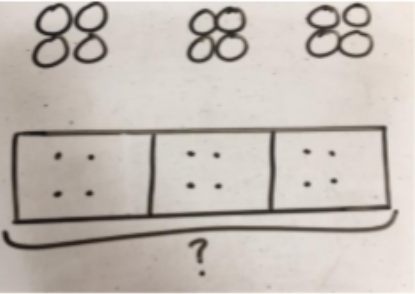
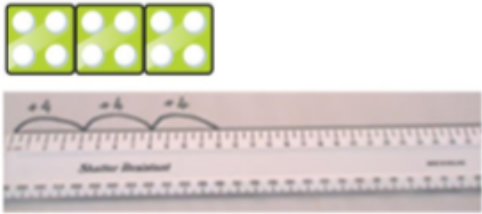
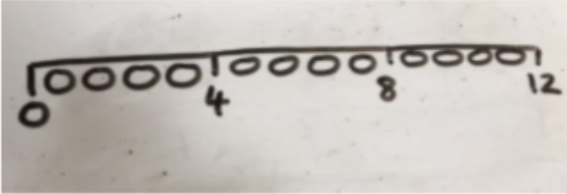
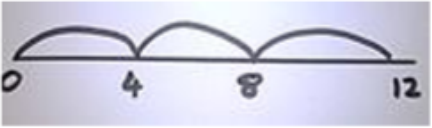
What is 186 less than 391?

Missing digit calculations

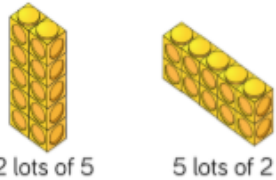
$$\begin{array}{r} 39\boxed{} \\ -\boxed{}\boxed{}6 \\ \hline \boxed{}05 \end{array}$$

Calculation policy: Multiplication

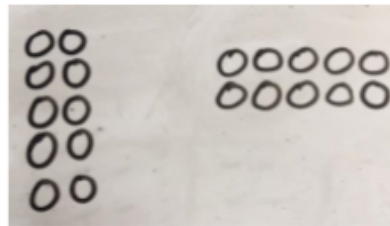
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>
<p>Number lines to show repeated groups- 3×4</p>  <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jumps of four.</p> <p>$3 \times 4 = 12$</p> 

Use arrays to illustrate commutativity counters and other objects can also be used.
 $2 \times 5 = 5 \times 2$



Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

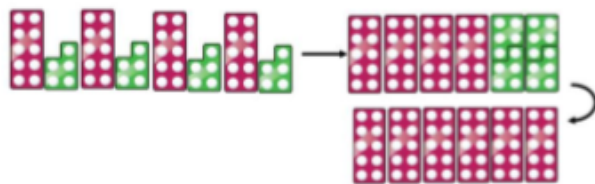
$$10 = 2 \times 5$$

$$5 \times 2 = 10$$

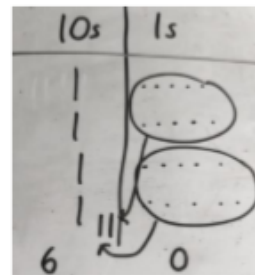
$$2 + 2 + 2 + 2 + 2 = 10$$

$$10 = 5 + 5$$

Partition to multiply using Numicon, base 10 or Cuisenaire rods.
 4×15



Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

$$4 \times 15$$

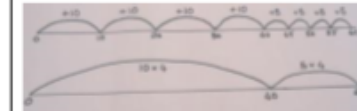
$$10 \quad 5$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

$$40 + 20 = 60$$

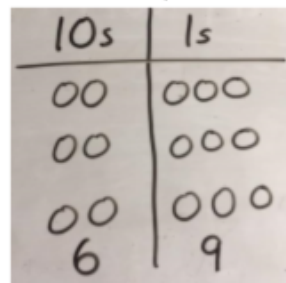
A number line can also be used



Formal column method with place value counters (base 10 can also be used.) 3×23

10s	1s
6	9

Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding.

$$3 \times 23 \quad 3 \times 20 = 60$$

$$20 \quad 3 \quad 3 \times 3 = 9$$

$$60 + 9 = 69$$

$$23$$

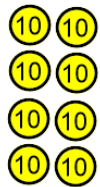
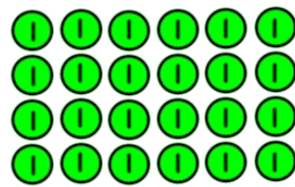
$$\times 3$$

$$\hline 69$$

Expanded method

Children will partition the largest number and multiply the tens by the multiplier using place value counters, they will then multiply the ones by the multiplier. Children will add both products together.

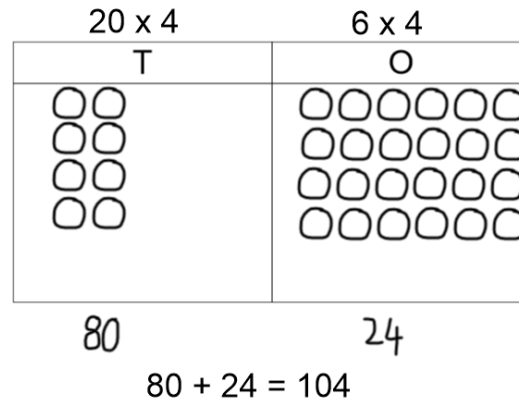
$$26 \times 4 = 104$$

 20×4

 80
 6×4

 24

$$80 + 24 = 104$$

Children will represent the counters pictorially. Children can draw this on a white board or in a place value chart.

$$26 \times 4 = 104$$

**Written method**

Children will partition the largest number into tens and ones then multiply by the multiplier. Writing the calculation at the side and the H T O above supports the children with place value.

2	6	\times	4						
				H	T	O			
					2	6			
				x		4			
					2	4	(6 x 4)		
				+	8	0	(2 0 x 4)		
				1	0	4			

Grid method

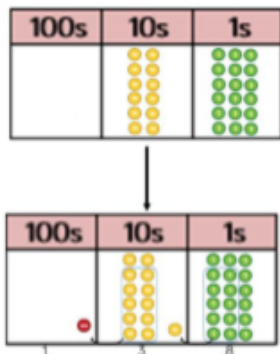
As above

Children partition the larger number into tens and ones. They multiply this by the multiplier in a grid. Model writing on H T O to support children with place value.

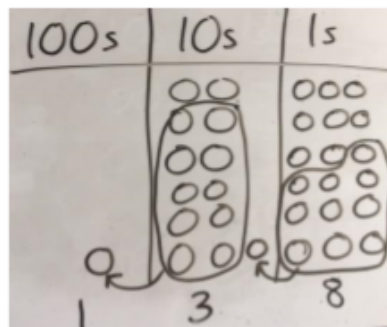
Written method

	26×4 <table style="margin: auto; border-collapse: collapse;"> <tr><td></td><td></td><td style="text-align: center;">T</td><td style="text-align: center;">O</td><td></td></tr> <tr><td></td><td style="text-align: center;">x</td><td style="border-right: 1px solid black; padding: 2px 5px;">20</td><td style="padding: 2px 5px;">6</td><td></td></tr> <tr><td></td><td style="text-align: center;">4</td><td style="border-right: 1px solid black; padding: 2px 5px;">(10)(10)</td><td style="padding: 2px 5px;">(1)(1)(1)(1)(1)(1)</td><td></td></tr> <tr><td style="text-align: center;">O</td><td></td><td style="border-right: 1px solid black; padding: 2px 5px;">(10)(10)</td><td style="padding: 2px 5px;">(1)(1)(1)(1)(1)(1)</td><td></td></tr> <tr><td></td><td></td><td style="border-right: 1px solid black; padding: 2px 5px;">(10)(10)</td><td style="padding: 2px 5px;">(1)(1)(1)(1)(1)(1)</td><td></td></tr> <tr><td></td><td></td><td style="border-right: 1px solid black; padding: 2px 5px;">(10)(10)</td><td style="padding: 2px 5px;"></td><td></td></tr> <tr><td></td><td></td><td style="border-right: 1px solid black; padding: 2px 5px;">80</td><td style="padding: 2px 5px;">24</td><td></td></tr> <tr><td></td><td></td><td colspan="2" style="text-align: center;">$80 + 24 = 104$</td><td></td></tr> </table>			T	O			x	20	6			4	(10)(10)	(1)(1)(1)(1)(1)(1)		O		(10)(10)	(1)(1)(1)(1)(1)(1)				(10)(10)	(1)(1)(1)(1)(1)(1)				(10)(10)					80	24				$80 + 24 = 104$			$26 \times 4 = 104$ <table style="margin: auto; border-collapse: collapse;"> <tr><td></td><td></td><td style="text-align: center;">T</td><td style="text-align: center;">O</td><td></td></tr> <tr><td></td><td style="text-align: center;">x</td><td style="border-right: 1px solid black; padding: 2px 5px;">20</td><td style="padding: 2px 5px;">6</td><td></td></tr> <tr><td style="text-align: center;">O</td><td style="text-align: center;">4</td><td style="border-right: 1px solid black; padding: 2px 5px;">80</td><td style="padding: 2px 5px;">24</td><td></td></tr> <tr><td></td><td></td><td colspan="2" style="text-align: center;">$80 + 24 = 104$</td><td></td></tr> </table>			T	O			x	20	6		O	4	80	24				$80 + 24 = 104$		
		T	O																																																											
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		$80 + 24 = 104$																																																												

Formal column method with place value counters.
 6×23



Children to represent the counters/base 10, pictorially
 e.g. the image below.



Formal written method

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 11
 \end{array}$$

When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .
 To get 2480 they have solved 20×124 .

$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 2480 \\
 \hline
 3224 \\
 \hline
 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6×23

23	23	23	23	23	23

?

Mai had to swim 23 lengths, 6 times a week.
 How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$


Find the product of 6 and 23

$$6 \times 23 =$$

$$\boxed{} = 6 \times 23$$

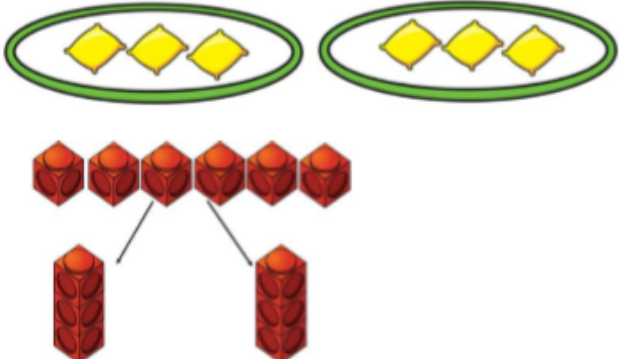
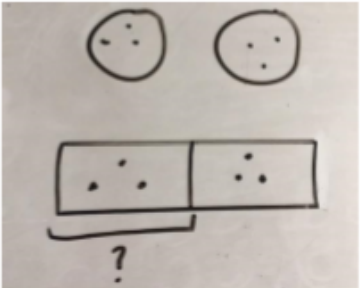

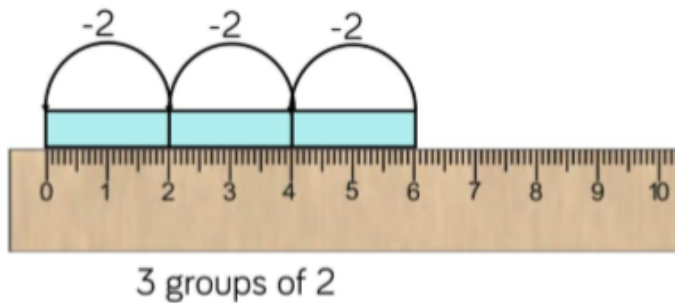
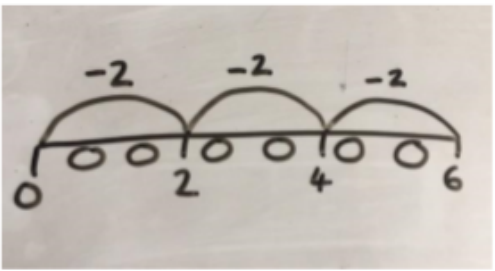
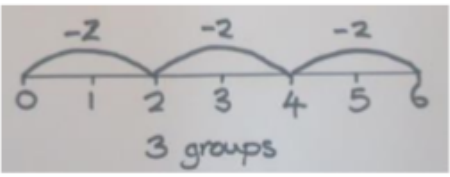
$$\begin{array}{r}
 6 \quad 23 \\
 \times \underline{23} \quad \times \underline{6} \\
 \hline \quad \hline
 \end{array}$$

What is the calculation?
 What is the product?

100s	10s	1s
		

Calculation policy: Division

Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
<p>Sharing using a range of objects. $6 \div 2$</p> 	<p>Represent the sharing pictorially.</p> 	<p>$6 \div 2 = 3$</p>  <p>Children should also be encouraged to use their 2 times tables facts.</p>
<p>Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$</p> 	<p>Children to represent repeated subtraction pictorially.</p> 	<p>Abstract number line to represent the equal groups that have been subtracted.</p> 

2d + 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

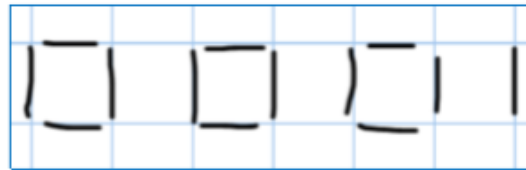
$$13 \div 4$$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

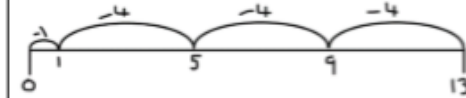


There are 3 whole squares, with 1 left over.

$$13 \div 4 = 3 \text{ remainder } 1$$

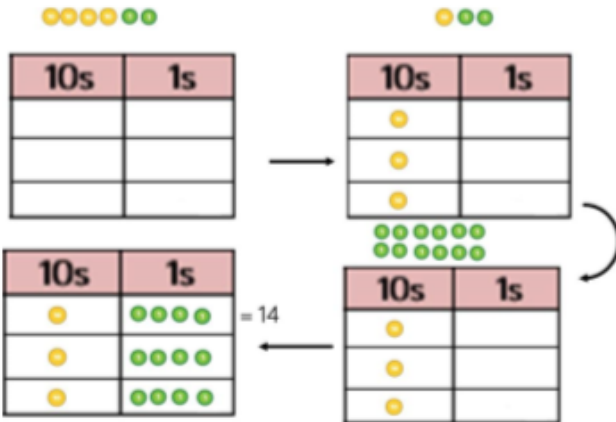
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

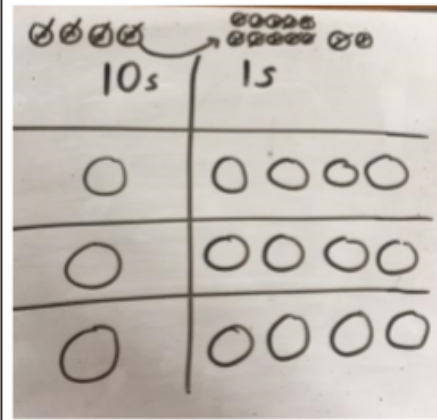


Sharing using place value counters.

$$42 \div 3 = 14$$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$42 \div 3$$

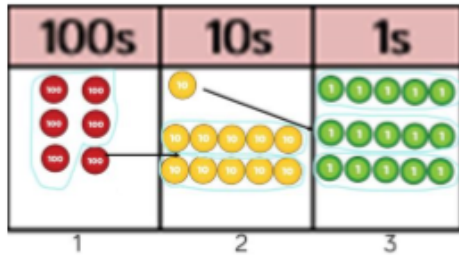
$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

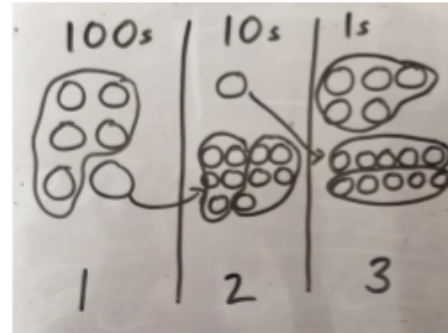
$$10 + 4 = 14$$

Short division using place value counters to group.
 $615 \div 5$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



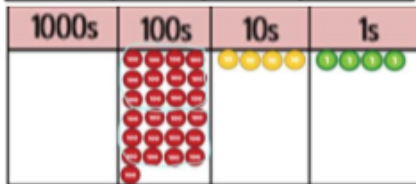
Children to do the calculation using the short division scaffold.

$$5 \overline{) 615} \begin{matrix} 123 \\ \underline{615} \\ 0 \end{matrix}$$

Long division using place value counters
 $2544 \div 12$

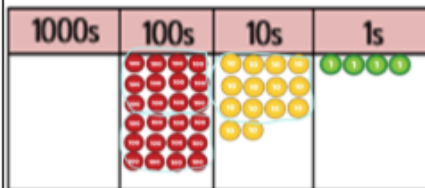


We can't group 2 thousands into groups of 12 so will exchange them.



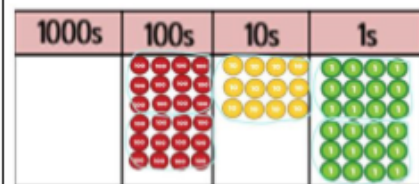
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$12 \overline{) 2544} \begin{matrix} 02 \\ \underline{24} \\ 1 \end{matrix}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?
What is the answer?



5. Mathematical Language

High expectations of the mathematical language used are essential, with staff only accepting what is correct. Consistency across the school is key:

Correct Terminology	Incorrect Terminology
ones	units
zero	(the letter o)
exchange exchanging regrouping	stealing borrowing
Bar Model	
Whole Part	
Known Unknown	